



# Neutrino Oscillation Phenomenology



Stephen Parke Fermilab June 1, 2004

- Solar (I-2) Sector
- Atmospheric (2-3) Sector
- (I-3) Sector
- CP Violation & Mass Hierarchy
- Conclusions

# Mixing Overview:

flavor

mass

$$|\nu_{\alpha}\rangle = U_{\alpha i}|\nu_{i}\rangle.$$

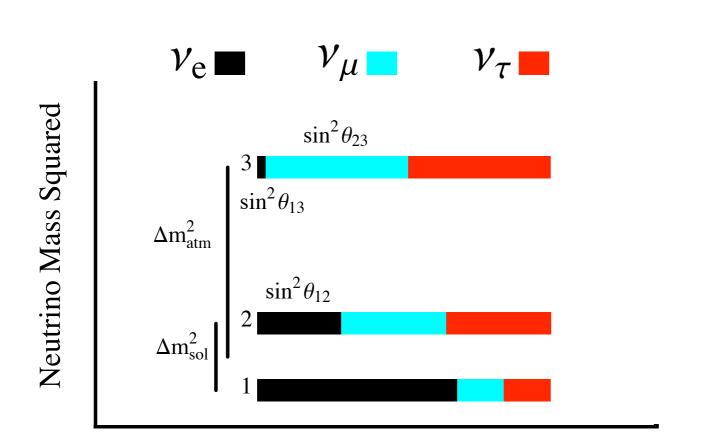
(using  $s_{ij} = \sin \theta_{ij}$  and  $c_{ij} = \cos \theta_{ij}$ )

$$U_{\alpha i} = \begin{pmatrix} 1 & & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ & -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} \\ & -s_{12} & c_{12} \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix}$$

Atmos. L/E  $\mu \to \tau$  Atmos. L/E  $\mu \leftrightarrow e$  Solar L/E  $e \to \mu, \tau$  Majorana

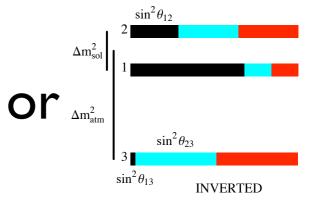
 $\sim$ 18 km/MeV

 $0\nu\beta\beta$ 



~500 km/GeV

central values  $\theta_{12}$ ,  $\theta_{23}$ max. for  $\theta_{13}$ and  $|\sin \delta| = 1$ 



Fractional Flavor Content

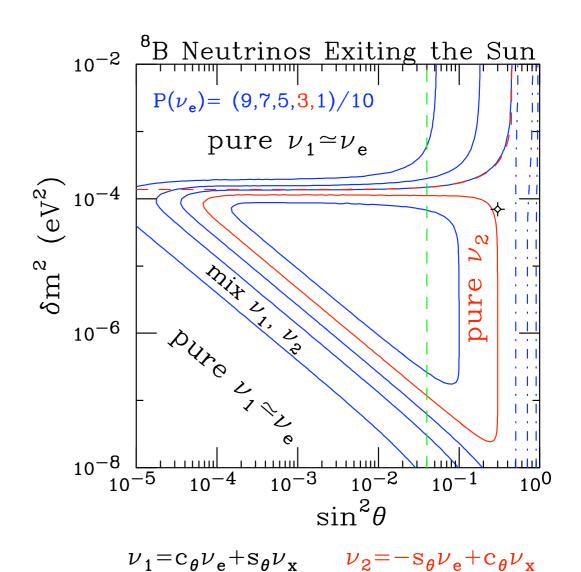
# Solar (I-2) Sector:

SNO, KamLAND, SK ...

$$\delta m_{21}^2 = +7.1 \pm 2.0 \times 10^{-5} eV^2$$
$$0.23 < \sin^2 \theta_{12} < 0.35$$

 $\sin^2 \theta_{12} \geq \frac{1}{2}$  excluded at > 5  $\sigma!$  Sign of  $\delta m^2_{21}$  determined at this C.L.

Due to matter effects the  $^8\text{B}$  solar neutrinos exit the sun as  $\nu_2$ .



$$P_{\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_x\right) \cos 2\theta_0 \cos 2\theta_N$$

SP. PRL 57,1275(1986)

$$r = N_1/(N_1 + N_2)$$

$$\frac{CC}{NC} = r\cos^2\theta_{12} + (1 - r)\sin^2\theta_{12}$$
$$= \sin^2\theta_{12} + r\cos 2\theta_{12}$$
$$\approx \sin^2\theta_{12}$$

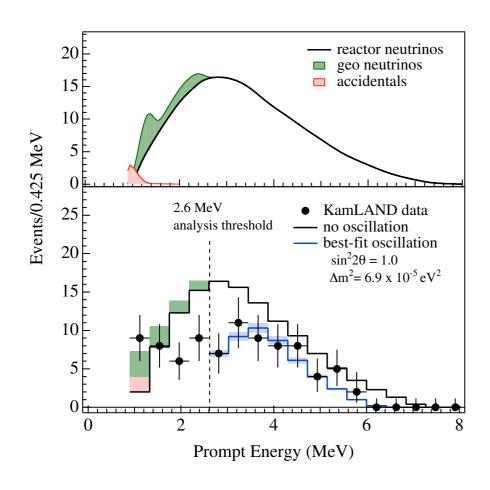
$$0.31 \pm 0.03 \approx 0.29 \pm 0.03$$



Thus SNO's  $\frac{CC}{NC}$  is a direct measure of  $\sin^2\theta_{12}$ .

(Up to small corrections.)

#### KamLAND Result:

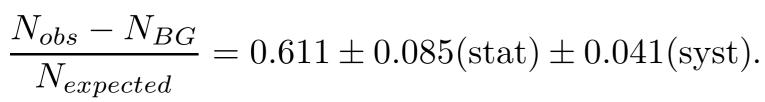


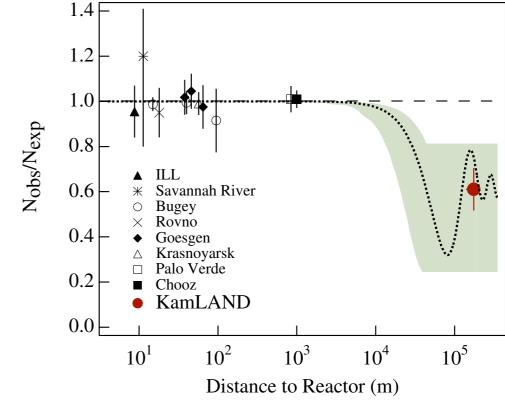
#### No spectral distortion seen:

$$P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta_{12}$$

Use SNO's  $\sin^2 \theta_{12}$ 

$$=1-2(0.3)(0.7)\approx0.6$$





#### Atmospheric (2-3) Sector:

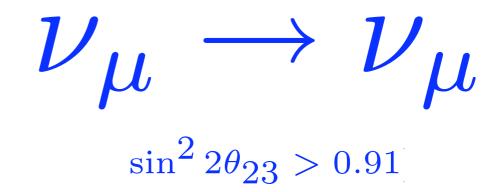
Neutrino Mass Squared

(23)-Sector: SK, K2K

$$|\delta m_{32}^2| = 1.9 - 3.0 \times 10^{-3} \ eV^2$$

$$0.35 < \sin^2 \theta_{23} < 0.65$$

(obtained from  $\sin^2 2\theta_{23} > 0.91$ )

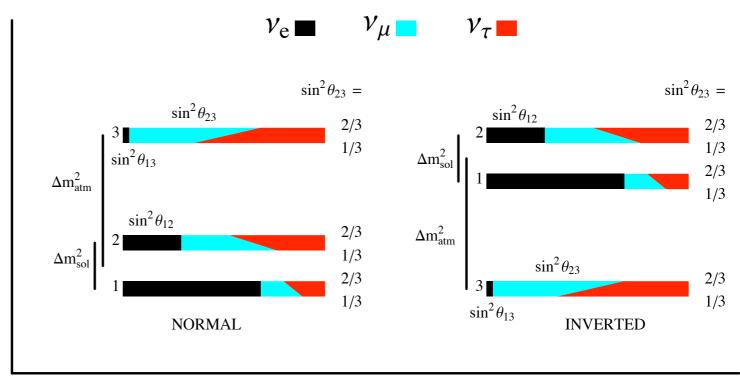


Magnitude of  $\delta m_{32}^2$  and  $\sin^2\theta_{23}$  both poorly known!

Sign of  $\delta m_{32}^2$  Unknown !!!

MINOS improves on

 $|\delta m_{32}^2|$ 



Fractional Flavor Content varying  $\sin^2 \theta_{23}$ 

O. Mena and SP hep-ph/0312312

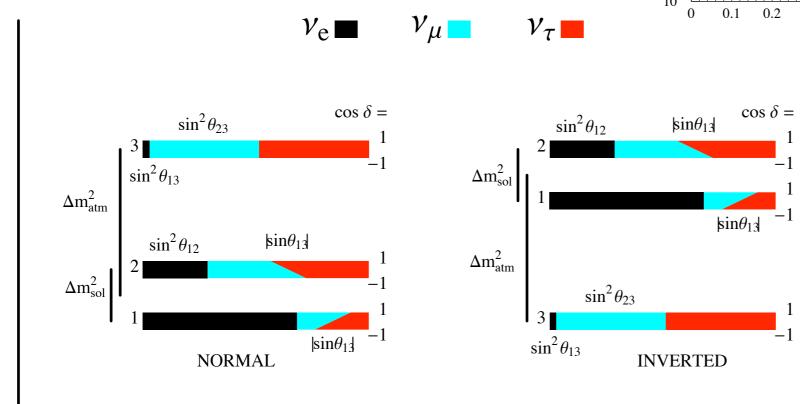
# (I-3) Sector:

#### Chooz, SK and K2K

 $\sin^2 \theta_{13} < 0.03 - 0.05$ limit  $|\delta m_{32}^2|$  dependent

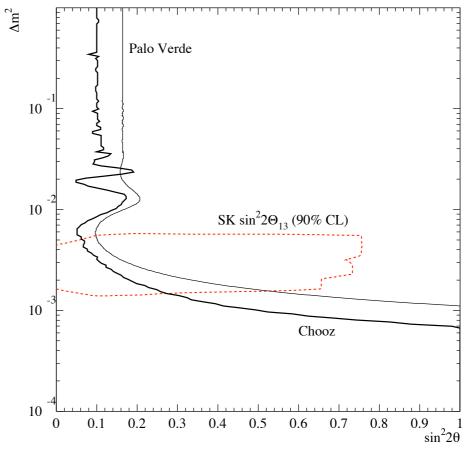
$$0 \le \delta_{CP} < 2\pi$$

Unknown!



#### Fractional Flavor Content varying $\cos \delta$

#### Minos to Chooz / 2



# Only insensitive to

sign of  $\sin \delta$ 

For  $\mu \Leftrightarrow \tau$  symmetry  $\theta_{23} = \pi/4$  and  $\delta = \pi/2$  or  $3\pi/2$  unless  $\theta_{13} \equiv 0$ 

Neutrino Mass Squared

#### Super-Chooz:

interest in Japan, Europe, Russia, USA (CA and IL), China ....

$$1 - P_{\nu_e \to \nu_e} = \sin^2 2\theta_{13} \left[ \sin^2 \Delta_{atm} + \mathcal{O} \left( \frac{\Delta_{solar}}{\Delta_{atm}} \right) \right] + \mathcal{O} \left( \frac{\Delta_{solar}}{\Delta_{atm}} \right)^2$$
>1%
<3%
<0.1%

kinematical phase

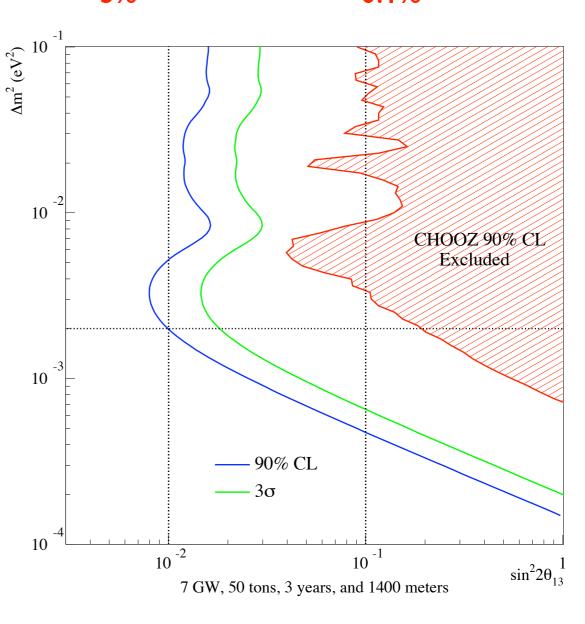
$$\Delta_{atm} = \frac{\delta m_{atm}^2 L}{4E} = 1.27 \frac{\delta m_{atm}^2 L}{E}$$

Clean measurement of

 $\sin^2 2\theta_{13}$  down to 0.01

Systematics limit experiment:

Could be "quick" and "cheap" but ...



J. Link, Columbia

#### Leptonic CP and T Violation in Oscillations

$$u_{\mu} \leftrightarrow \nu_{e} \qquad \Longleftrightarrow \qquad \bar{\nu}_{\mu} \leftrightarrow \bar{\nu}_{e} \qquad \qquad \text{Super-Beams}$$



$$\nu_e \leftrightarrow \nu_\mu \qquad \Longleftrightarrow \qquad \bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$$



$$\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$$

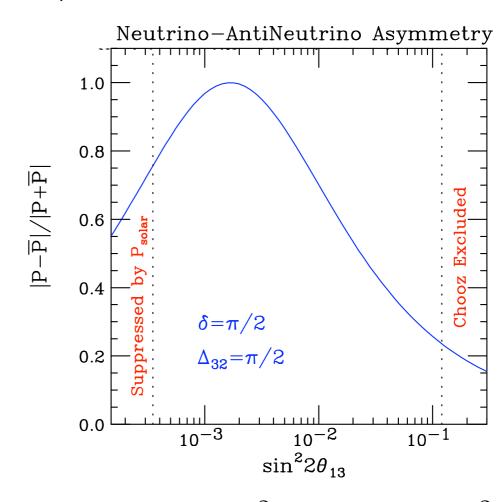
**Nu-Factory** 

$$P_{\nu_{\mu} \to \nu_{e}} = |a_{\mu \to e}^{atm} + a_{\mu \to e}^{sol}|^{2}$$

CP Violation comes from the Difference in the Interference of  $a_{\mu \to e}^{atm}$  amd  $a_{\mu \to e}^{sol}$ for neutrinos verses anti-neutrinos.

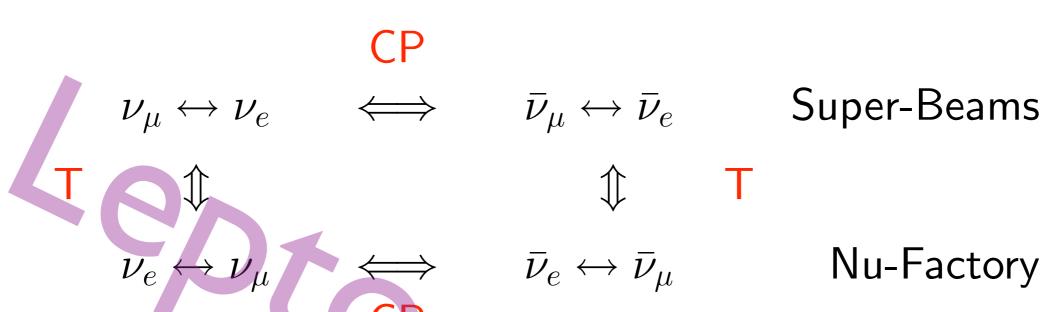
#### CAN BE LARGE!!!.

Important parameters are  $\theta_{13}$  and  $\delta$ .



$$\Delta_{ij} = \frac{\delta m_{ij}^2 L}{4E} = 1.27 \frac{\delta m_{ij}^2 L}{E}$$

#### Leptonic CP and T Violation in Oscillations

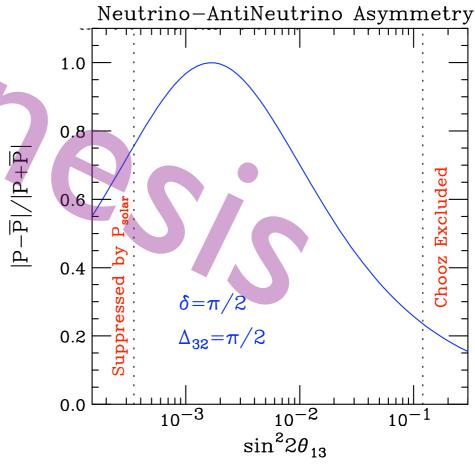


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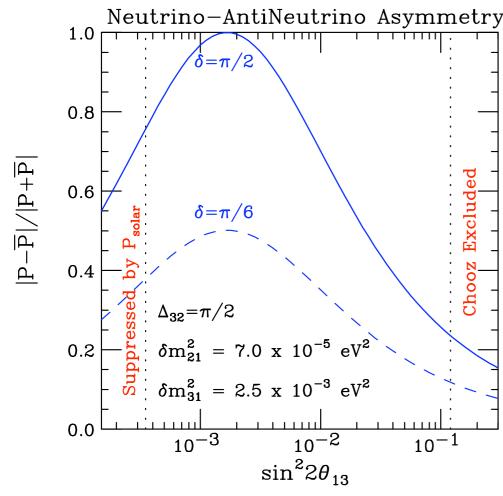
$$\Delta_{ij} = \frac{\delta m_{ij}^2 L}{4E} = 1.27 \frac{\delta m_{ij}^2 L}{E}$$

$$P^{atm}(\nu_{\mu} \to \nu_{e}) = |a^{atm}|^{2} = \sin^{2}\theta_{23}\sin^{2}2\theta_{13}\sin^{2}\Delta_{31}$$

$$P^{sol}(\nu_{\mu} \to \nu_{e}) = |a^{sol}|^{2} = \cos^{2}\theta_{23}\cos^{2}\theta_{13}\sin^{2}2\theta_{12}\sin^{2}\Delta_{21}$$

### relative phase is $\Delta_{32} \pm \delta$

$$P(\nu_{\mu} \to \nu_{e}) = |a^{atm} + a^{sol}|^{2} = P^{atm} + P^{sol} + 2\sqrt{P^{atm} \cdot P^{sol}} \cos(\Delta_{32} \pm \delta)$$

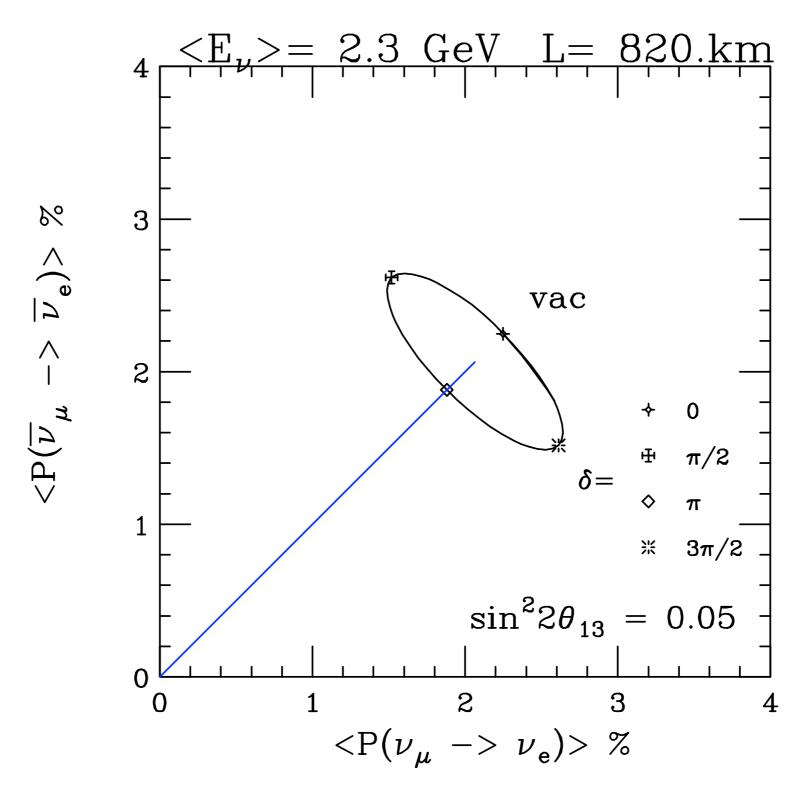


#### Maximum Asymmetry when

$$|a^{atm}| = |a^{sol}|$$

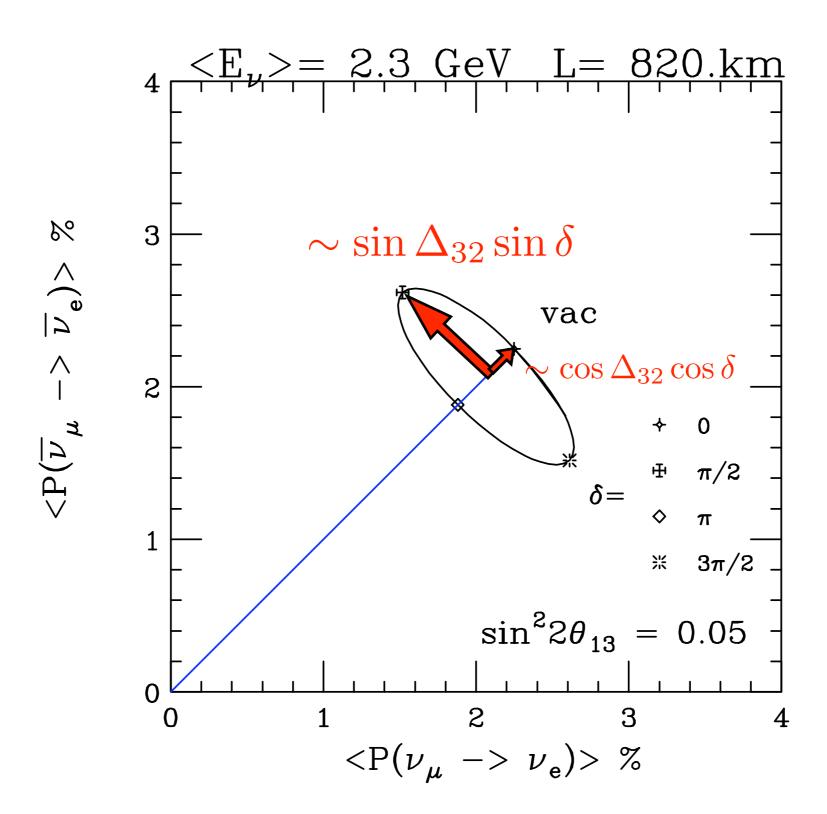
$$\sin^2 2\theta_{13} \approx \frac{\sin^2 2\theta_{12}}{\tan^2 \theta_{23}} \left[ \frac{\pi}{2} \frac{\delta m_{21}^2}{\delta m_{31}^2} \right]^2$$

$$\approx 0.002$$



#### Bi-Probability Plots:

 $\begin{array}{c} \mbox{Minakata and Nunokawa} \\ \mbox{hep-ph/}0108085 \end{array}$ 

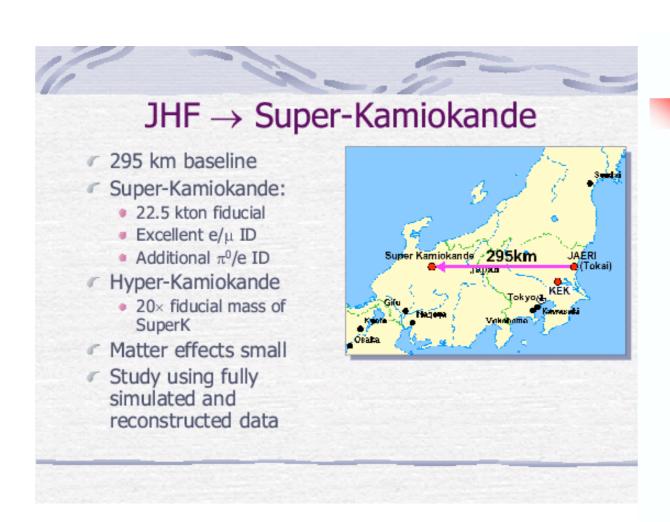


#### Bi-Probability Plots:

 $\begin{array}{c} {\rm Minakata~and~Nunokawa} \\ {\rm hep-ph/0108085} \end{array}$ 

### Off-Axis Beams:

**BNL 1994** 



The NUMI Beamline

Two functionally identical neutrino detectors

Mixilian MI Fermilab 10 km Soudan 730 km 12 km

Det. 1

Det. 2

E, (GeV)

On axis

14 mrad off-axis

L=295 km and Energy at Vac. Osc. Max. (vom)  $E_{vom} = 0.6~GeV \left\{ \frac{\delta m_{32}^2}{2.5\times 10^{-3}~eV^2} \right\}$ 

Energy near 2 GeV  $E_{vom} = 1.8 \ GeV \left\{ \frac{\delta m_{32}^2}{2.5 \times 10^{-3} \ eV^2} \right\} \times \left\{ \frac{L}{820 \ km} \right\}$ 

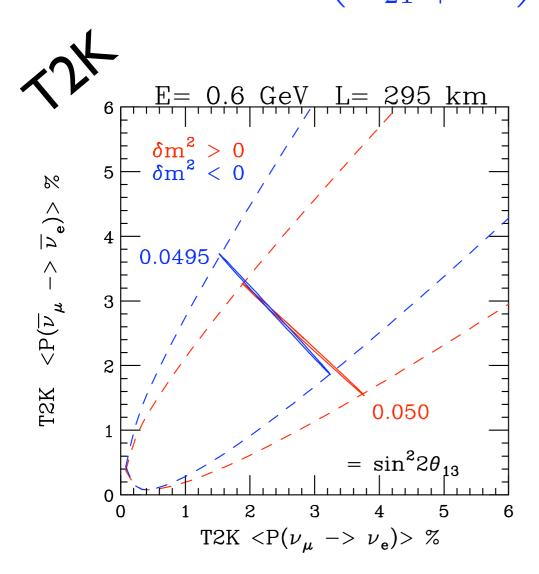
L=700 - 1000 km and

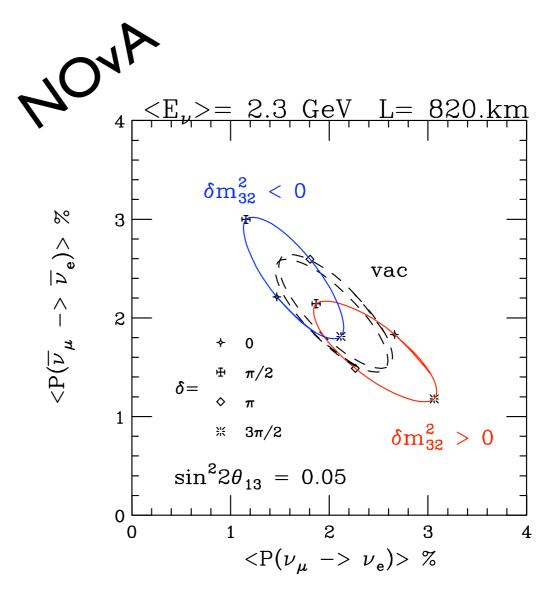
#### Matter Effects:

$$a = G_F N_e / \sqrt{2} = (4000 \ km)^{-1}$$

$$\sin \Delta_{31} \implies \left(\frac{\Delta_{31}}{\Delta_{31} \mp aL}\right) \sin(\Delta_{31} \mp aL)$$

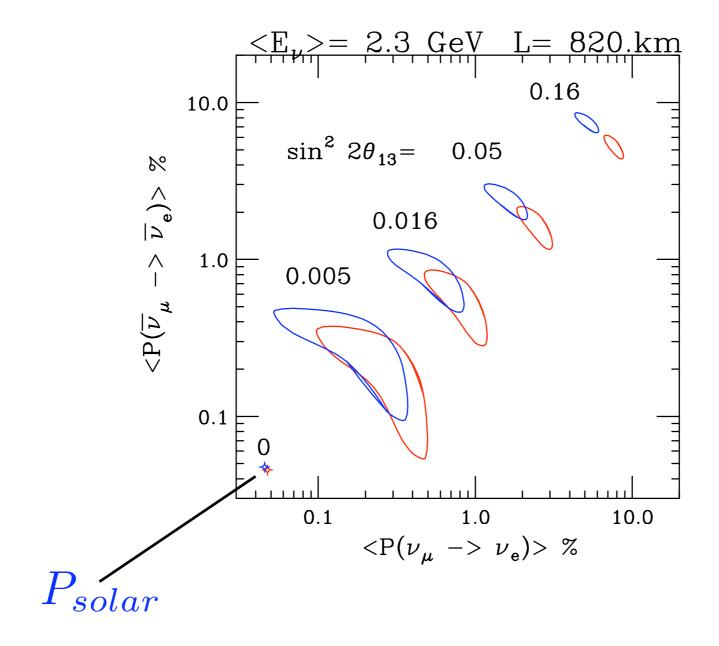
$$\sin \Delta_{21} \implies \left(\frac{\Delta_{21}}{\Delta_{21} \mp aL}\right) \sin(\Delta_{21} \mp aL) \approx \Delta_{21}$$





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# Varying $\sin^2 2\theta_{13}$ Log scale:



# Two Signs:

 $sign of \delta m_{31}^2$ 

normal v inverted hierarchy

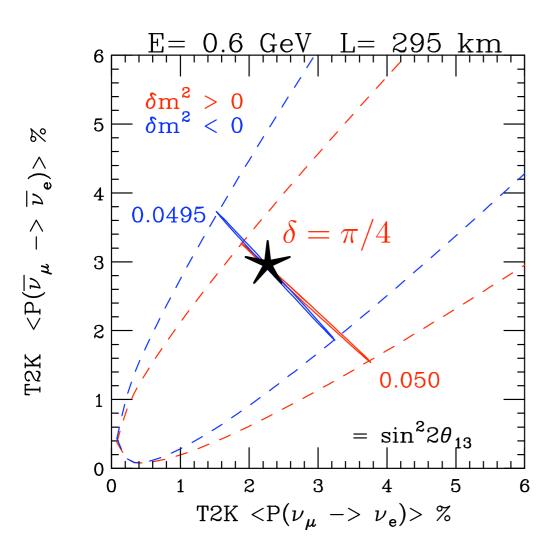
and

Leptonic CP Violation



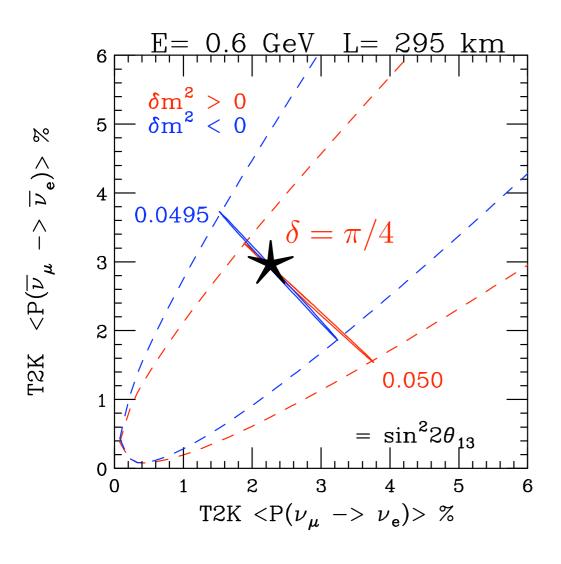
O. Mena and SP

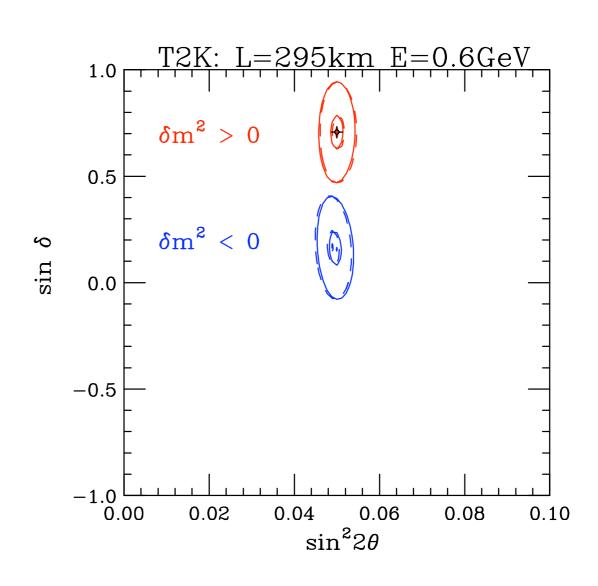
# **T2K:**



# **T2K:**

$$\sin \delta_{+} = \sin \delta_{-} + 0.5 \sqrt{\frac{\sin^{2} 2\theta_{13}}{0.05}}$$

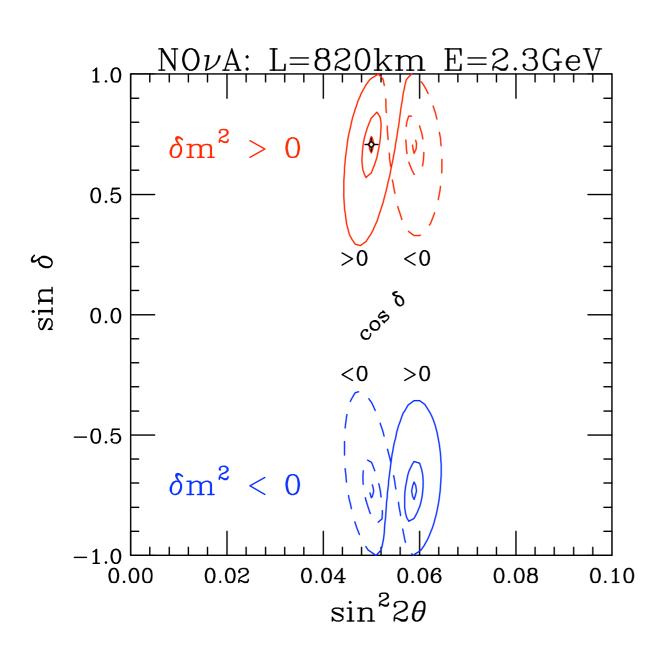


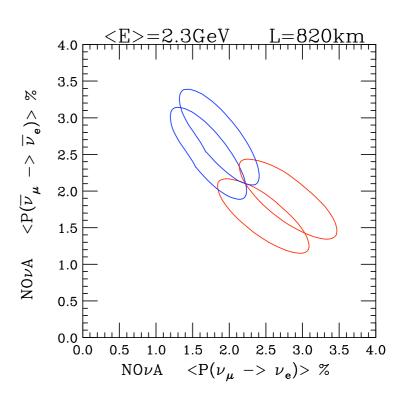


no info on sign of  $\cos \delta = \pm \sqrt{1 - \sin^2 \delta}$ 

# NOvA:

$$\sin \delta_{+} = \sin \delta_{-} + 1.5\sqrt{\frac{\sin^{2} 2\theta_{13}}{0.05}}$$

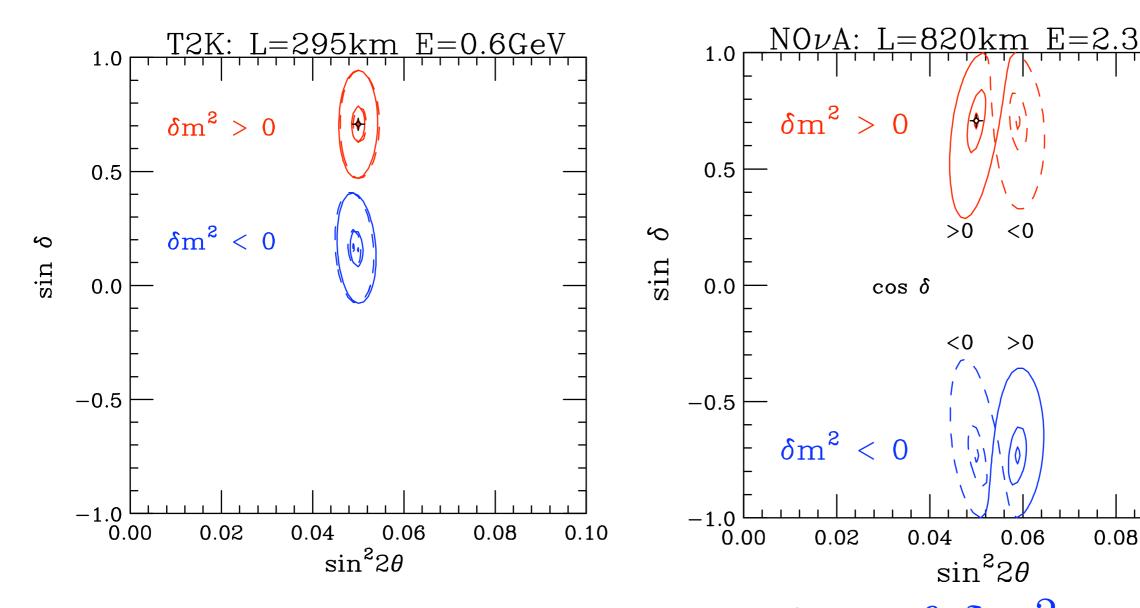




sensitive to sign of

$$\cos \delta = \pm \sqrt{1 - \sin^2 \delta}$$

# T2K + NOvA



with sufficient events
T2K plus NOvA determines

sign of  $\delta m_{31}^2$  =hierarchy

0.10

#### Hierarchy: T2K Nu v. NOvA Nu

 $\langle P(\nu_{\mu} -> \nu_{e}) \rangle$ 

NuMI

#### sign $\delta m_{32}^2$

At Vac. Osc. Max.,  $\Delta_{32} = \frac{\pi}{2}$ 

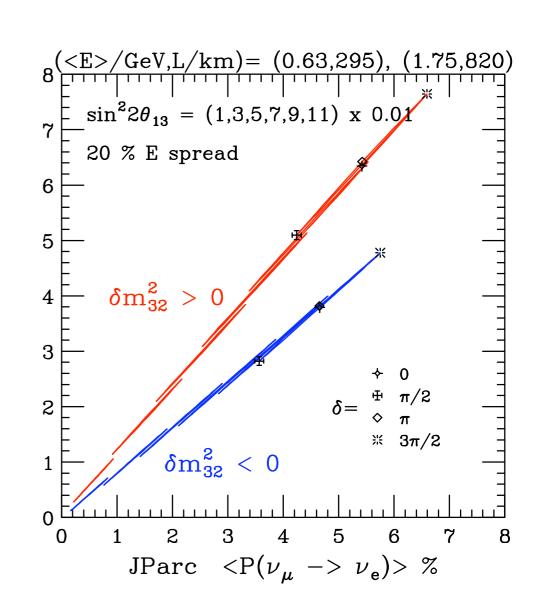
$$P_{mat} = \left(1 \pm 2 \frac{E}{E_R}\right) P_{vac}$$
 where  $E_R \simeq 12$  GeV.

Therefore, if NuMI and JParc both run Neutrinos at Vac. Osc. Max.

$$P_N = \left(1 \pm 2 \frac{(E^N - E^J)}{E_R}\right) P_J$$

i.e. 
$$P_N \approx (1.2 \text{ or } 0.8)P_J$$

Need about 100 events in each expt.



Separation degraded for  $E^N > E_{vom}$ .

Minakata, Nunokawa and SP – hep-ph/0301210

### Conclusions:

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• \sin^2 2\theta_{13}: Can be measured by Reactor Exp.(\sim 0.01), Long Baseline Exp.(\sim 0.005), Nu Factories (\sim 10^{-4}).
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- sign of  $\delta m_{31}^2$  and  $\sin \delta$ :  $\Rightarrow$  Mass Hierarchy and CP Violation.  $\nu_{\mu} \rightarrow \nu_{e}$  Superbeam Long Baseline Exp. running BOTH  $\nu$  and  $\bar{\nu}$ .
- $\theta_{23}$ : To break the  $\theta_{23} \leftrightarrow \frac{\pi}{2} \theta_{23}$  degeneracy. Combination of Reactor and Long Baseline Exps.  $\sin^2 2\theta_{13}$  v.  $\sin^2 \theta_{23} \sin^2 2\theta_{13}$ .

sparkE HQ&L 04 24

If the size of  $\theta_{13}$  is in range of the LBL experiments,  $\sin^2 2\theta_{13} \geq 0.005$ , then a few carefully choosen counting experiments with sufficient accuracy can determine  $\theta_{13}, \quad \delta_{CP}, \quad \text{sign of } \delta m_{31}^2, \quad \theta_{23}.$ 

A Fabulous Opportunity in the Neutrino Osc. Sector!!!

Leaving the Questions of: Majorana v Dirac?, Steriles? and Absolute Mass Scale,  $M_{lite}$ ?

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